**Infinity**

* Define infinity
  + As a class, point, limit and concept
* Look at examples of infinity occurring in everyday and introduce ideas of different types of infinity
  + Idea of the finite and infinite having no determinable gap between
* Aleph and Cantor cardinality
  + Tortoise and Hare
  + Lines and points
  + Cantor set
* Aleph and Cantor arithmetic

Infinity is a concept that is present in all facets of everyday life, whether we realise its influence and involvement or not. But despite its consistent appearance, very few of us fail to understand it. Intuition and initial thoughts of the concept of the infinite develop as we grow, from the biggest number we can think of as a kid, to the appreciation of its unbounded continuation. As a concept, there are a number of ways to define infinity, before we discuss how infinity is present in everyday, and its use in maths.

Infinity can be defined simply as that without bound, continuing indefinitely. In mathematics, infinity can be used to count the number of elements of specific types of sets, despite not being a real number. Infinity can also refer to classes, where an infinite set is defined as having the same cardinality as at least one of the proper or strict subsets of the set. This differs from the finite arithmetic definition of a set, where the entire set has a greater cardinality than any of its proper or strict subsets. Finally, infinity may be defined as a point, using the Riemann Sphere, which relies on the process of stereographic projection, to map points on the plane to points on the sphere.

In the real world, there are a few easily noticeable infinities that we encounter on a daily basis but may not realise it. Take first a simple one metre ruler. Between the two ends of the ruler there exist an infinite number of measurements and positions. Laying the ruler down on the floor, we are still able to walk from the start of the ruler to the end, despite moving through an infinite number of positions on the ruler. These type of confusing thought problems, especially to do with the motion of two objects travelling at different velocities, prompted further research and development of theories by the mathematicians Cantor and Zeno.

Another crucial observation about infinities is that there are different orders, or different sizes of infinities, a fact discovered by Cantor. Take the set of all integers. We know this to be infinite. Consider now the set of all even integers. Again, this is an infinite set. Yet we can construct a one-to-one correspondence between the set of all integers and the set of even integers. Thus, we are able to “count” both the set of all integers and similar sets that can be placed in the same one-to-one correspondence with the set of integers. This is known as countably infinite, and has cardinality aleph-null. However, now consider the set of all real numbers. Cantor proved that there is no one-to-one mapping of the set of integers to the set of real numbers. Thus, the set of real numbers has a greater cardinality, C, also known as an uncountable infinite set. Thus, we have seen two types of infinity that have different “sizes”, despite the fact infinity is said to be unbounded and the “forever”. There exist higher cardinalities of infinity, but their connection has yet to be seen.

The final observation, or concepts to consider is the Cantor set. This is a set constructed by taking the line interval from 0 to 1 and removing the middle third segment. The process is then repeated on all remaining intervals an infinite number of times. One would think this leaves an empty set, but rather, a set containing the same number of points as the original line segment, but no interval of length greater than zero. Again, this goes back to our discussion of the infinite as a set, where the whole has no greater cardinality than some of the proper subsets, and the removal of elements does not affect the cardinality of the infinite set.